

A Reduced Form Model of Default Spreads with Markov Switching Macroeconomic Factors

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Outline of the presentation

- 1 Motivation
- 2 Pricing framework and formulas
 - Framework
 - Formulas
- 3 Estimation and calibration
 - Corporate spreads
 - Markov switching parameters
 - Preference parameters
 - Default probability parameters
- 4 Empirical results

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Motivation

- Corporate yield spreads:
 - Default, liquidity, taxes
- Empirical findings:
 - Proportion of default in spread is small
- Recent research directions
 - Changes in regime and interest rates (Bansal and Zhou (2002), Ang, Bekeart and Wei (2007))
 - Links between spreads and macroeconomic variables (Tang and Yang (2005), Amato and Luisis (2006), Alexander (2007))

This study

- Blends these two recent research directions :
 - Markov switching dynamics for the factors
 - Risk factors: changes in consumption and inflation
- Closed form analytical approximations:
 - Risk-free zero-coupon bond prices
 - Risky zero-coupon bond prices
 - Term structure of default probabilities
- Empirical study:
 - Out of sample
 - Proportion of the yield spread caused by **default risk**
 - Links of default spread with inflation and consumption

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Pricing framework

- Stochastic discount factor:

$$M_{t,t+1} = \exp(\ln \beta - \gamma c_{t+1} - \pi_{t+1})$$

- Risky bond prices:

$$V(t, n) = E_t [M_{t,t+1} \times (1 - Lh_{t+1}) \times V(t + 1, n - 1)]$$

- Growth rate of consumption and inflation are Markov switching:

$$c_t = a_{s_t^c}^c + b_{s_t^c}^c c_{t-1} + e_t^c$$

$$\pi_t = a_{s_t^\pi}^\pi + b_{s_t^\pi}^\pi \pi_{t-1} + e_t^\pi$$

- Conditional default prob. affine in cons. growth and inflation:

$$h_{t+1} = \alpha_{s_{t+1}} + \alpha_{s_{t+1}}^c c_{t+1} + \alpha_{s_{t+1}}^\pi \pi_{t+1}$$

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Pricing formulas

- Recursive formulas for risk-free bond prices, risky bond prices and survival probabilities :

$$\exp(A - B^c c_t - B^\pi \pi_t)$$

- Analytical approximations
- Conditional on the state of the Markov Switching
- $A(\bullet)$, $B^c(\bullet)$ and $B^\pi(\bullet)$:
 - obtained recursively
 - functions of the Markov Switching parameters
 - functions of the preference parameters
 - functions of the default prob. parameters
 - functions of the constant recovery rate L

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Corporate spreads

- Lehman Brothers Fixed Income data base (1986-1996)
- Category: Baa, Industrial
- Estimation : Nelson and Siegel (1987) approach

Average of estimated spreads

	2 years	5 years	10 years
Baa industrial	0.0116	0.0123	0.0118

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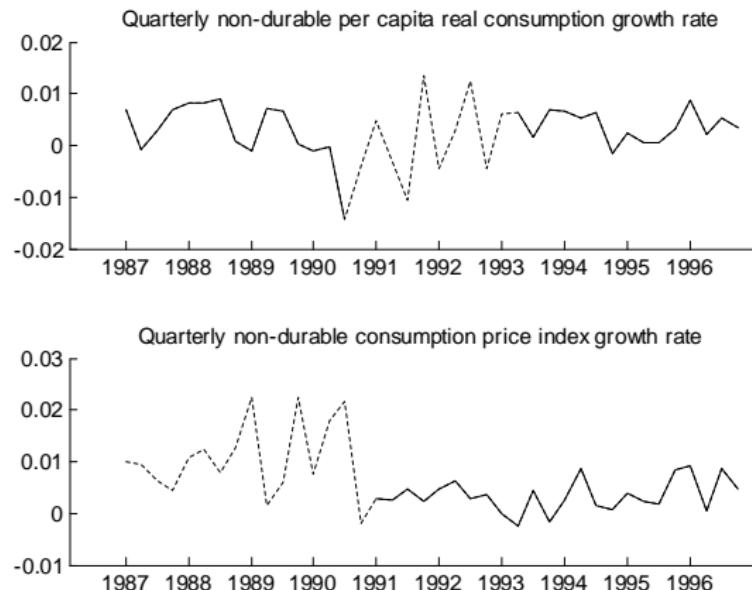
Preference parameters

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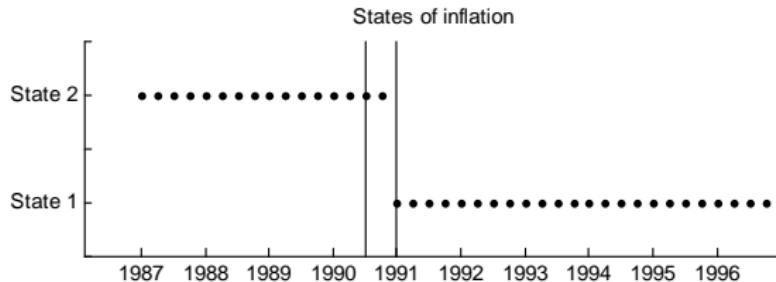
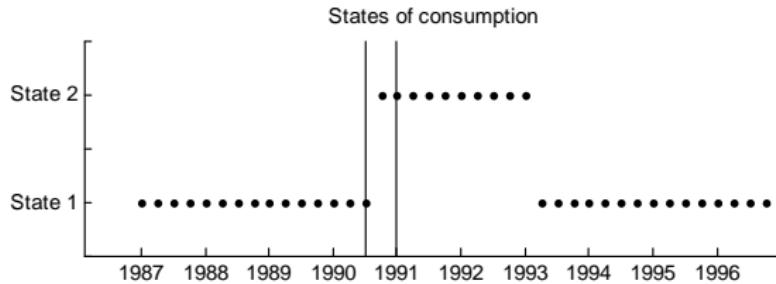
Markov switching parameter values

- Consumption growth and inflation



Markov switching parameter values

- MLE estimation from 57:I to 96:IV



Markov switching parameter values

- MLE estimation from 57:I to 96:IV

$$c_t = \begin{cases} \mathbf{0.0029} + 0.1617 c_{t-1} + \mathbf{0.0036} \varepsilon_t^c & \text{if } s_{c,t} = 1 \\ \mathbf{0.0028} + \mathbf{0.3298} c_{t-1} + \mathbf{0.0091} \varepsilon_t^c & \text{if } s_{c,t} = 2 \end{cases}$$

$$\pi_t = \begin{cases} \mathbf{0.0040} + 0.0690 \pi_{t-1} + \mathbf{0.0040} \varepsilon_t^\pi & \text{if } s_{\pi,t} = 1 \\ \mathbf{0.0063} + \mathbf{0.5912} \pi_{t-1} + \mathbf{0.0074} \varepsilon_t^\pi & \text{if } s_{\pi,t} = 2 \end{cases}$$

$$\phi^c = \begin{pmatrix} 0.88 & 1 - 0.88 \\ 1 - 0.89 & 0.89 \end{pmatrix}, \quad \phi^\pi = \begin{pmatrix} 0.97 & 1 - 0.97 \\ 1 - 0.96 & 0.96 \end{pmatrix}$$

$$\rho_{11} = -0.13, \quad \rho_{12} = -0.37, \quad \rho_{21} = -0.12, \quad \rho_{11} = -0.58$$

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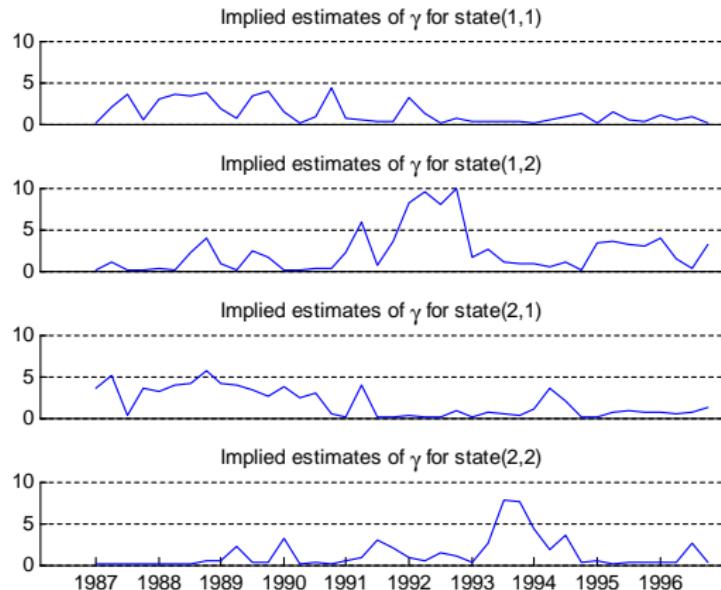
Preference parameters

Default probability parameters

4 Empirical results

Preference parameters estimation

- Data: risk-free yield curves
- Implied preference parameter at **each quarter**
- Estimates of risk aversion parameters (γ 's)



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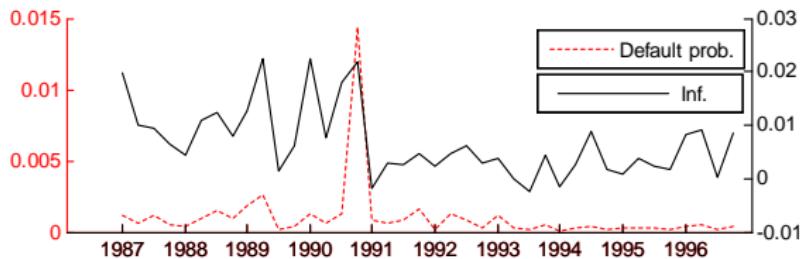
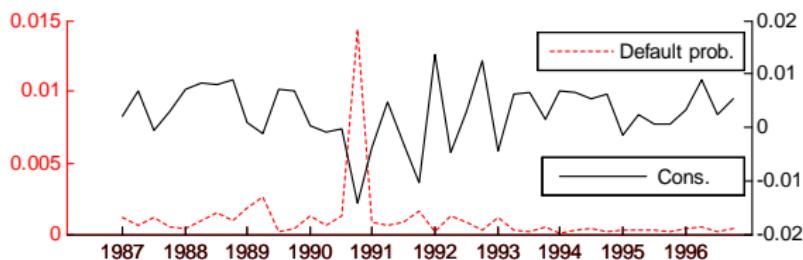
Default probability parameters

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Default probability parameters

$$h_{t+1} = \alpha_{s_{t+1}} + \alpha_{s_{t+1}}^c c_{t+1} + \alpha_{s_{t+1}}^\pi \pi_{t+1}$$

- Estimation of a term structure of default probability for Baa
- Find α 's allowing a close fit with the observed term structure



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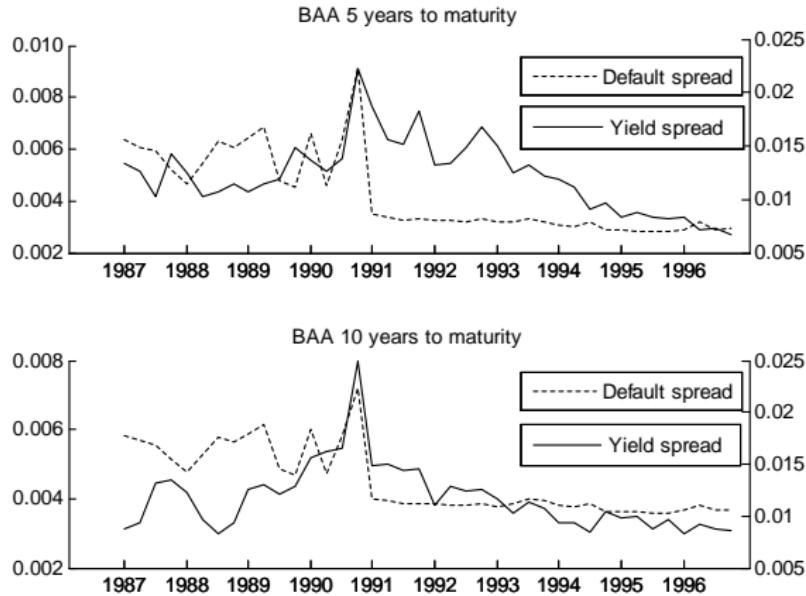
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Default spreads



Default spreads (estimated out of sample)

- 5 years to maturity Baa:

	$s_t = (1, 1)$	$s_t = (1, 2)$	$s_t = (2, 1)$	$s_t = (2, 2)$
average DS_t/YS_t	33%	48%	21%	41%
Corr. DS_t with c_t	0.61	-0.35	-0.06	—
Corr. DS_t with π_t	-0.02	0.84	-0.82	—